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ON THE PHOTOGRAPHIC BRIGHTNESS OF THE FIXED STARS.

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The investigations relating to the photographic brightness of the fixed stars contained in this paper were made with the aid of an equatorially mounted DALLMEYER portrait-lens of $6^{in}.05$ aperture, loaned to this Observatory by the U. S. N. Observatory for the purpose of photographing the next total solar eclipse at Cayenne, South America.

Professor HOLDEN placed this instrument in my charge, and requested me to make a series of experiments on atmospheric absorption of the light, and on the photographic brightness, of the fixed stars, so that the extended work of the same character which it is intended to execute in South America could be more intelligently and profitably performed.

The photographic focus was carefully determined by making several series of short exposures, and trails, of bright stars both inside and outside of the adopted position of the plate. The position of the plate for each setting was read off on a scale which I cut on the tube. All the exposures were made on 5×7 Seed 26 plates.

Leaving the work relating to atmospheric absorption to a future paper, let us consider the subject of the photographic brightness of stars as determined by the dimensions of their circular images on the sensitive plate. (As the dimensions—widths—of the trails could only be accurately determined for the brighter stars, I finally avoided examining trails for this special investigation.)

As the whole subject was comparatively new to me, several weeks were spent in work of an experimental character. A careful study of the data given on the exposed plate was made with the aid of our excellent measuring engine. I finally came to the conclusion that the diameter of the image of an "over-exposed" star could be used

to determine the star's brightness with accuracy. To find the law of variation of the diameter of the photographic image for a variation of both the aperture of the objective and the time of exposure, seven different stops, varying in diameter from 5.41 inches to 1.91 inches, were used, and exposures of 1^s, 2^s, 4^s, 8^s, 16^s, 32^s, 64^s, and 128^s duration made for each stop. In order to be sure of the effective aperture of the stops, they were placed centrally in front of the objective, and not in the usual place between the lenses. The diameters of these stops, which we will number 1, 2, 3, etc., are as follows:

No.	1	2	3	4	5	6	7
Diameter	in. 5.41	in. 4.59	in. 3.81	in. 3.31	in. 2.72	in. 2.31	in. 1.91

The figures in the following table give the diameters of the images of *Polaris*, in inches, as measured on one of the plates:

POLARIS.

Exposure Times.	Diameters of Image for different Stops and Times.						
	1	2	3	4	5	6	7
1^s	in. 0.0048	in. 0.0049	in. 0.0049	in. 0.0048	in. 0.0045	in. 0.0041	in. 0.0036
2	58	57	59	52	47	45	37
4	68	66	58	59	57	55	40
8	78	74	70	72	40	—	48
16	81	—	72	64	56	52	50
32	92	72	74	76	65	61	53
64	116	90	91	86	78	68	59
128	139	117	102	96	84	78	67

I tried to represent these numbers by various equations of the second and third degrees as functions of the aperture and time, but finally concluded that they could only be represented by an equation of the n^{th} degree, or, in other words, that the logarithms of the variables enter into the equation. I then made a similar set of exposures, using *α Lyræ* (discussed further on), and found that the function which represented the diameters was of precisely the same form. I

have deduced the following general expression for the diameter of the photographic image of a star:

$$d = \alpha + \beta . \log D + \gamma . D . \log t \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In which, for a given star,

d = the measured diameter of the photographic image;
α = a constant depending only on the sensitive plate and the atmospheric state;
β = " " " " " " " "
γ = " " " " " " " "
D = the effective diameter of the objective (stop);
t = the time of exposure expressed in seconds.

In order to determine the most probable values of α, β and γ for a particular case it will be more convenient to place

$$\alpha + \beta \log D = a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\gamma D = b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (1) then becomes

$$d = a + b \log t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In selecting the unit for D it must be remembered that with a small stop the images, for comparatively short exposures, are small and faint. Greater accuracy may therefore be expected from large apertures. I have accordingly chosen six inches (6^{in}) as the unit of *D*. The diameters of the stops in terms of this unit are therefore as given below:

Stop.	1	2	3	4	5	6	7
Diameter	0.902	0.765	0.635	0.552	0.453	0.385	0.318
Log. of Diam.	-0.045	-0.116	-0.197	-0.258	-0.344	-0.415	-0.498

Equation (1) shows that when $t = 1^s$ and $D = 6^{in}$, we have $d = a$; in other words, a is the diameter of the photographic image of the star for an aperture of six inches and an exposure time of one second.

Taking *Polaris* for the standard star, the above-measured diameters give the following values for a and b as found by the method of least squares; each equation of condition being of the form:

$$d = a + b \log t$$

Stop.	a	b
	in.	in.
1	0.0051	0.0032
2	53	22
3	51	19
4	46	22
5	43	15
6	43	13
7	36	12

To find the value of α and β we have the following equations of condition

$$
\begin{aligned}
\alpha - 0^{in}.045\,\beta &= 0^{in}.0051 \\
\alpha - \phantom{0^{in}}.116\,\beta &= \phantom{0^{in}}.0053 \\
\alpha - \phantom{0^{in}}.197\,\beta &= \phantom{0^{in}}.0051 \\
\alpha - \phantom{0^{in}}.258\,\beta &= \phantom{0^{in}}.0046 \\
\alpha - \phantom{0^{in}}.344\,\beta &= \phantom{0^{in}}.0043 \\
\alpha - \phantom{0^{in}}.415\,\beta &= \phantom{0^{in}}.0043 \\
\alpha - \phantom{0^{in}}.498\,\beta &= \phantom{0^{in}}.0036
\end{aligned}
$$

The solution of which by the method of least squares gives for values of α and β

$$\alpha = 0^{in}.0055 \qquad \beta = 0^{in}.0033$$

The diameter of the photographic image of *Polaris* for six inches aperture and one second exposure is therefore, for this particular case, $0^{in}.0055$.

The independent values of γ given by the expression $\gamma = \frac{b}{D}$ are

b	D	γ			
in.		in.			
0.0032	0.902	0.0035	from 8 different exposures	with Stop	1
22	.765	0029	" 7 ditto		2
19	.635	0030	" 8 ditto		3
22	.552	0040	" 8 ditto		4
15	.453	0033	" 8 ditto		5
13	.385	0034	" 7 ditto		6
12	.318	0038	" 8 ditto		7

Taking the mean of the values of γ, we have for the images of *Polaris* the equation

$$d = 0^{\text{in}}.0055 + 0.0033 \log D + 0.0034\, D \log t \quad . \quad . \quad (5)$$

The residuals obtained by subtracting the diameters computed by the above formula from the measured diameters are as follows:

POLARIS.

Exposure Time.	Observation—Computation.						
	1	2	3	4	5	6	7
	in.	in.	in.	in.	in.	in.	in.
1^s	-0.0005	-0.0002	0.0000	+0.0001	+0.0001	0.0000	-0.0003
2	- .0005	- .0002	+ .0004	.0000	- .0001	.0000	- .0004
4	- .0004	- .0001	- .0004	+ .0001	+ .0004	+ .0006	- .0005
8	- .0003	- .0001	+ .0002	+ .0008	——	——	.0000
16	- .0009	——	- .0003	- .0005	- .0004	- .0005	- .0002
32	- .0008	(0018)	- .0007	+ .0001	.0000	.0000	- .0001
64	- .0007	- .0008	+ .0003	+ .0006	+ .0008	+ .0003	+ .0001
128	- .0020	- .0011	+ .0008	+ .0010	+ .0010	+ .0009	+ .0006

The diameters of the images of *α Lyræ* on a plate exposed Sept. 2, are as follows:

α LYRÆ.

Exposure Time.	Diameters of Images for different Stops and Times.						
	1	2	3	4	5	6	7
1^s	0.0093	0.0088	0.0073	0.0061	0.0055	0.0046	0.0045
2	114	91	—	70	63	55	48
4	123	96	—	80	70	--	57
8	148	107	102	109	80	71	66
16	* —	125	114	114	88	—	72
32	* —	146	133	122	104	—	83
64	* —	169	151	145	119	—	—

*** In this table, as in the one for *Polaris*, the missing figures belong to cases in which the images, on account of imperfect pointing, are not circular but elongated; while for stop 1 the images are so close together that the larger ones overlap, and, consequently, were not used.**

The equation which fairly represents these diameters is:

$$d = 0^{\text{in}}.0070 + 0^{\text{in}}.0050 \log D + 0.0074 \, D \log t \quad . \quad . \quad (6)$$

the individual values of γ, found by dividing each b by the corresponding D, are:

b	$\div$ D	$= \gamma$
0.0067	$\div$ .902	= 0.0074
46	.765	.0060
42	.635	.0066
46	.552	.0083
34	.453	.0075
30	.385	.0078
27	.318	.0085

The observed values of d, minus the values computed by equation (6), are as given below:

a Lyræ.

Exposure Time.	Observation—Computation.						
	1	2	3	4	5	6	7
1^s	+0.0005	(+0.0024)	+0.0013	+0.0004	+0.0002	-0.0002	0.0000
2	+ .0006	.0010	——	+ .0001	.0000	- .0001	- .0004
4	- .0005	- .0002	——	- .0001	- .0003	——	- .0002
8	——	- .0008	.0000	+ .0015	- .0003	- .0002	.0000
16	——	- .0007	- .0002	+ .0008	- .0005	——	- .0001
32	——	- .0003	- .0001	- .0004	+ .0001	——	+ .0003
64	——	+ .0003	+ .0006	- .0009	+ .0006	——	——

From equation (6) we infer that, for six inches aperture and one second exposure time, the diameter of *a Lyræ's* image on this particular plate is $0^{\text{in}}.0070$. Comparing equation (6) with equation (5) we learn that the increase in the diameter of the image of *a Lyræ* on this plate for any t is 2.2 times as rapid as it is in the case of *Polaris* for the same t on the plate first described; so that, if other things were equal, the difference between the photographic energy of two stars could be more accurately determined from comparatively long exposures than from short ones. (The *rate* of increase, of course, varies inversely as t.)

Now, let

$$d = \alpha_0 + \beta_0 \log D + \gamma_0 \, D \log t \quad . \quad . \quad . \quad . \quad (7)$$

be the equation giving the diameters for a particular star taken as a standard, and let

$$d' = \alpha + \beta \log D_0 + \gamma D_0 \log t \quad . \quad . \quad . \quad . \quad (8)$$

be the equation which gives the diameter of the image of *any* star for the *constant aperture* D_0 (unity $\doteq 6^{in}$); then if Q represents the particular aperture in equation (7) which, for the same value of t makes $d = d'$, the reciprocal of this quantity, or $\frac{1}{Q}$, substituted in place of D_0 must, for all values of t, satisfy equation (8) for $d' = d$ if the assumed law* is theoretically exact. Q, then, becomes a measure of the square root of the relative brightness of the two stars, since, if we assume that the amount of energy required to produce a given impression on a given plate is always the same, whatever the unit of energy (intensity) may be, the total amount of energy for the same telescope can be considered as varying directly with the area of the aperture, or with D^2. Hence, if B_0 and B denote respectively the brightness of the standard and comparison stars, we can at once write:

$$\left(\frac{Q}{D_0}\right)^2 = \frac{B}{B_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Q being that value of D which when substituted in the equation for the standard star (equation 7) will make $d = d'$. In other words, *Q is the diameter of the aperture which the standard star would require to produce, in the time t, an image having the same diameter as that of any other star photographed with an aperture* D_0 (= six inches) *in the same time t.* Equation (7) can therefore be written:

$$d' = \alpha_0 + \beta_0 \log Q + \gamma_0 Q \log t \quad . \quad . \quad . \quad . \quad (11)$$

Let us now take the equations deduced from the measured diameters of the images of *Polaris* and *α Lyræ*, and see to what degree of accuracy the necessary theoretical relations between Q and D_0 will represent the observed data. For $d = d'$ we have the equation:

$$0.0055 + 0.0033 \log Q + 0.0034\, Q \log t = 0.0070 + 0.0050 \log D_0 + 0.0074\, D_0 \log t$$

After a few trials, for different values of t, we obtain the approximate value $D_0 = 0.48$ when $Q = 1$, and, according to the above considerations, we should also have $Q = 2.10$ when $D_0 = 1$. Both of these conditions should be fulfilled for all values of t.

As the images of the two stars are on different plates I have not thought it worth while to derive a more accurate relation between Q and D_0 for this particular case.

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* The law expressed in equations (1) and (7).

The approximate relation will, however, serve to show the agreement between our theory and the data derived from actual observations.

The accompanying table contains the computed values of d and d' for each t, for reciprocal values of Q and D_0:

EXPOSURE TIME.	$D_0 = 1.00$ $Q = 2.10$		$D = 0.48$ $Q = 1.00$	
	Polaris d	*a Lyræ* d'	*Polaris* d	*a Lyræ* d'
1^s	0.0070	0.0066	0.0055	0.0054
2	.0087	.0092	.0065	.0065
4	.0109	.0114	.0075	.0075
8	.0130	.0140	.0086	.0086
16	.0151	.0159	.0096	.0097
32	.0173	.0181	.0106	.0107
64	.0195	.0204	.0117	.0118

If the two sets of star images had been impressed upon the same plate, we would have inferred the photographic brightness of *a Lyræ* to be about 4.4 times that of *Polaris.** As, however, the diameters of the star images on different plates taken from the same box are not always the same for equal exposures, it became necessary to make a separate investigation covering this particular phenomenon.

I found that if we express the diameters of the image of *Polaris* on any plate in terms of the diameters given on the plate for which equation (5) holds good (which we will call the standard plate), we have only to multiply the second member of equation (5) by such a number x that for a given t the measured d will be satisfied. From a series of comparisons I find that x is practically constant for the different values of t. The general equation for any No. 26 Seed plate exposed in the stellar focus of the particular telescope used in these investigations I therefore assume to be

$$\frac{d}{x} = 0^{in}.0055 + 0^{in}.0033 \ (\log Q + Q \log t) \quad . \ . \ . \quad (12)$$

since, for all practical purposes, β_0 and γ_0 in equation (5) are the same.

* Neglecting atmospheric absorption.

To find the value of Q from this equation we can write:

$$\log Q + Q \log t = \log (Q t^Q) = \frac{d}{0.0033 \,.\, x} - 1.67 \qquad (13)$$

In order, however, to facilitate the determination of Q for certain observed values of d and t, I have computed the following table, by means of which Q can be obtained by simple interpolation.

The horizontal argument is Q, the vertical argument is t, and the tabular function corresponding to these arguments is the measured d of equation (13) for $x = 1.00$.

Exposure Time.	Q.									
	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20	3.60	4.00
2^s	0.0046	0.0060	0.0070	0.0078	0.0085	0.0091	0.0098	0.0103	0.0109	0.0114
4	.0050	.0068	.0081	.0093	.0105	.0115	.0125	.0135	.0145	.0154
8	.0054	.0078	.0093	.0109	.0125	.0139	.0153	.0167	.0181	.0194

If for a particular plate any measured diameter is d, the argument for entering the above table is $x.d$, and x is to be taken as a constant for the same plate.

We will now give a few examples illustrating the application of the formulas for determining the brightness of the fixed stars:

Example I. On September 5th, 1889, *Polaris*, α *Aurigæ*, γ *Cephei* and α *Tauri* were photographed on the same plate with exposures of 2^s, 4^s and 8^s. The measured diameters are:

Exposure Time.	Measured Diameters (= d).			
	Polaris.	α *Aurigæ.*	γ *Cephei.*	α *Tauri.*
2^s	0.0064	0.0081	0.0042	0.0060
4	.0074	.0090	.0052	.0061
8	.0080	.0109	.0056	.0072

The aperture being six inches for all the exposures, we first assume the plate to be a standard one, and find d with the argument Q = 1, either by means of equation (5) or, by interpolation, from the table:

POLARIS.

EXPOSURE TIME.	COMPUTED d.	O - C.
2^s	0.0065	- 0.0001
4^s	.0074	0.0000
8^s	.0085	- 0.0005

As the measured values are slightly smaller than those given by our assumed standard plate, we give x such a value that the (O - C) quantities will nearly balance each other. Placing $x = 1.03$ and multiplying the observed values of d by 1.03, the residuals (O - C) become respectively +0.0001, +0.0001, and - 0.0003.

To obtain the value of Q, by direct computation, for any star whose image is on this particular plate, we would therefore use the equation

$$\log Q + Q \log t = \frac{d}{0.0034} - 1.67 \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

in which d is the measured diameter corresponding to the time t.

The tabular values at once give the desired quantities by interpolation, first multiplying each measured d by 1.03 for the argument:

EXPOSURE TIME.	VALUES OF $Q = \sqrt{B}$.			
	Polaris.	*α Aurigæ.*	*γ Cephei.*	*α Tauri.*
2^s	1.04	1.89	0.32	0.84
4^s	1.05	1.60	0.32	0.69
5^s	0.90	1.68	0.47	0.73
Mean	1.00	1.72	0.42	0.75

Using the mean values of Q, we obtain the following residuals:

EXPOSURE TIME.	OBSERVATION - COMPUTATION.			
2^s	0.0000	+0.0003	- 0.0003	+0.0003
4	+ .0002	- .0004	+ .0002	- .0003
8	- .0003	- .0002	+ .0003	- .0001

If we use the familiar expression for the light-ratio of visual magnitudes,

$$B = (0.4)^{m-1}$$

(in which B and m are respectively the visual brightness and visual magnitude of any star) for expressing also the light-ratio for the photographic magnitudes m', we can write

$$m' = 1 - \frac{\log.(\kappa \cdot Q^2)}{\log.0.4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

in which κ is a constant depending upon the photographic magnitude of the standard star. For the purpose of comparing the photographic with the visual magnitudes, let us take *Polaris* as the standard star, and assume its photographic magnitude to be 2.00; equation (17) becomes

$$m' = 2 - \frac{\log Q^2}{0.4}$$

The values of m', for certain values of Q, can be taken from the accompanying table, which I have computed for illustration:

Q	0.00	0.20	0.40	0.60	0.80
0.00		5.49	3.99	3.11	2.48
1.00	2.00	1.60	1.27	0.98	0.72
2.00	0.50	0.29	0.10	-0.07	-0.24
3.00	-0.39	-0.53	-0.66	-0.78	-0.90

From an inspection of the table and the values of Q given in the next example, we see at once that if we wish to avoid negative numbers for expressing some observed magnitudes we must either represent the magnitude of *Polaris* by a greater number or change the light-ratio.

I have tabulated the photographic magnitudes of the four stars, together with the probable errors. The visual magnitudes as given in Volume XIV, *Harvard College Observatory Annals*, and the differences between the photographic and visual magnitudes, are also added:

Star.	Photog. Mag.	Probable Error.	Visual Mag.	Vis. - Photog.
Polaris	2.02	±0.11	2.2	+0.2
α Aurigæ	0.82	0.11	0.2	-0.6
γ Cephei	4.20	0.28	3.4	-0.8
α Tauri	2.62	0.13	1.0	-1.6

EXAMPLE II. During the night of September 6, in bright moonlight, I made exposures of 2^s, 4^s and 8^s on *Polaris*, *α Lyræ*, *α Cygni*, *α Aquilæ*, and four hours later the same plate was exposed on *α Pisces Australis*, *β Ceti*, *α Aurigæ*, *α Arietis* and *α Andromedæ*. The measured diameters are tabulated below. For this plate we see at once that the differences between the observed and computed values of *d* are such that the (C – O) values (+0.0001, – 0.0003 and +0.0003) practically balance each other; hence we place $x = 1.00$ and use equation (13) (or the table) to obtain the values of Q given below. For the (O – C) values the computed quantities are obtained by substituting the mean values of Q in equation (12):

EXPOSURE TIME.	MEASURED VALUES OF *d*.								
	Polaris.	*α Lyræ.*	*α Cygni.*	*α Aquilæ.*	*α Pis. Aust.*	*β Ceti.*	*α Aurigæ.*	*α Arietis.*	*α Androm.*
2^s	0.0066	0.0103	0.0092	0.0090	0.0077	0.0049	0.0075	0.0053	0.0079
4	.0072	.0134	.0116	.0106	.0087	.0054	.0094	.0063	.0090
8	.0088	.0169	.0136	.0129	.0099	.0060	.0106	.0066	.0116

EXPOSURE TIME.	RESULTING VALUES OF $Q = \sqrt{B}$.								
2^s	1.04	3.20	2.46	2.33	1.55	0.49	1.45	0.60	1.66
4	0.92	3.16	2.44	2.04	1.40	0.49	1.63	0.69	1.50
8	1.07	3.26	2.31	2.11	1.35	0.50	1.52	0.60	1.77
	1.01	3.21	2.30	2.15	1.43	0.49	1.53	0.63	1.64

EXPOSURE TIME.	THE MEAN VALUES OF Q, GIVE THE RESIDUALS,								
2^s	+0.0000	0.0000	+0.0002	+0.0003	+0.0002	0.0000	–0.0002	–0.0001	0.0000
4	+ .0001	– .0001	+ .0003	– .0002	.0000	.0000	+ .0003	+ .0003	– .0004
8	– .0003	+ .0002	.0000	– .0001	– .0003	+ .0001	+ .0001	– .0001	+ .0005

The numbers expressing the photographic brightness of each star in terms of that of *Polaris* are therefore, in the above order, 10.3, 5.3, 4.6, 2.0, 0.2, 2.3, 0.4 and 2.7. The only star of the list which was near to the zenith at the time its image was formed on the photographic plate is *α Lyræ;* the effect of moonlight, atmospheric

absorption and haze would therefore be at a minimum for this star, and its relative brightness would apparently be near a maximum. The same remarks apply to the results given in the next table as in the last example:

Star.	Photog. Mag.	Probable Error.	Visual Mag.	Vis. - Photog.
Polaris	+ 1.95	± 0.05	+ 2.2	+ 0.2
α *Lyræ*	— 0.53	± 0.02	+ 0.2	+ 0.7
α *Cygni*	+ 0.10	± 0.03	+ 1.5	+ 1.4
α *Aquilæ*	+ 0.34	± 0.06	+ 1.0	+ 0.7
α *Pis. Aust.*	+ 1.22	± 0.05	+ 1.3	+ 0.1
β *Ceti*	+ 3.53	± 0.01	+ 2.1	— 1.4
α *Aurigæ*	+ 1.08	± 0.05	+ 0.2	— 0.9
α *Arietis*	+ 3.01	± 0.07	+ 2.0	— 1.0
α *Androm.*	+ 0.93	± 0.07	+ 2.1	+ 1.2

No corrections for absorption, etc., have as yet been applied to the above results, which consequently refer to the apparent magnitude at the instant of exposure. The last column of the above table plainly shows that we can make no definite *a priori* estimate as to what the photographic magnitude of a star is if we simply know its visual magnitude. There is therefore no advantage (as Professor Holden has pointed out in his "Memorandum" to the Paris Photographic Conference) in following the methods used for visual magnitudes.

It is evident that we must first know the law of atmospheric absorption of the photographic rays before we can determine the true relative brightness of the stars; since each observed brightness requires a certain plus correction, depending directly upon the zenith distance, to reduce it to the brightness which would have been obtained at the zenith. Or each observed brightness could be reduced to what it would be at a certain zenith distance, as, for instance, that of the celestial pole at a given place. The photographs already taken show that this correction is quite sensible, even at small zenith distances. From some preliminary reductions I find that for this Observatory (altitude 4209 feet) the absorption of stellar

photographic brightness at 80° zenith distance is considerably more than fifty per cent. of the brightness reduced to 0° zenith distance. After a complete series of observations bearing upon this subject has been obtained at sea-level near the earth's equator, I hope to give, in a more or less complete state, the photographic magnitudes of a large number of the brighter stars in both hemispheres. Just how far down the scale of magnitudes the formulæ will hold good I am, as yet, unable to say.

In photographing faint stars the exposure time should evidently be so long as to make the diameters of the disks as great or greater than the faint penumbral image which, in the telescope used, surrounds the primitive umbral image in short exposures on faint stars; when this precaution is taken, it seems that the formulæ give consistent results, judging from a few experimental exposures. This form of image for short exposures on faint stars may, of course, be peculiar to this particular telescope. Too much stress cannot be laid upon the statement, *that if reliable results are to be obtained*, the objective must be of the first order of excellence and *the plate must be kept exactly in the stellar focus.*

Throughout this whole discussion I have purposely avoided bringing in any relation between aperture and focal length, as it seems probable that different telescopes must be compared before any definite conclusions can be drawn.

The results contained in the present paper are only to be considered as preliminary to a much more extended investigation to be undertaken in South America under the auspices of this Observatory, made possible through the generosity of Col. CROCKER.

In conclusion, I wish to express my obligations to Professor HOLDEN, Director of this Observatory, for his readiness in placing at my disposal everything which could in any way aid me in past and future investigations; for his practical help and advice relating to a subject which has claimed his attention for some time past, and which is destined to become the most important method of investigation in our science, viz: *Astronomical Photography.*

I also wish to thank Mr. BURNHAM for his kind and willing assistance in the photographic work.

J. M. SCHAEBERLE.

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